

On the effects of vortex trapping on the velocity statistics of tracers and heavy particle in turbulent flows

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The Lagrangian statistics of heavy particles and of fluid tracers transported by a fully developed turbulent flow are investigated by means of high resolution direct numerical simulations. The Lagrangian velocity structure functions are measured in a time range spanning about three decades, from a tenth of the Kolmogorov timescale, τ_η , up to a few large-scale eddy turnover time. Strong evidence is obtained that fluid tracers statistics are contaminated in the time range $\tau \in [1 : 10]\tau_\eta$ by a bottleneck effect due to vortex trapping. This effect is found to be significantly reduced for heavy particles which are expelled from vortices by inertia. These findings help in clarifying the results of a recent study by *H. Xu et al.* [Phys. Rev. Lett. **96** 024503 (2006)], where differences between experimental and numerical results on scaling properties of fluid tracers were reported.

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Suspensions of dust, droplets, bubbles, and other finite-size particles advected by incompressible turbulent flows are commonly encountered in many natural phenomena and applied processes ranging from cloud formation to industrial mixing [1, 2]. Understanding their statistical properties is thus of primary importance. From a theoretical point of view, the problem is more complicated than in the case of fluid tracers, point-like particles with the same density of the carrier fluid: inertia is responsible for the appearance of correlations between the particle positions and the structure of the underlying flow. It is well known indeed that heavy particles are expelled from vortical structures, while light particles tend to concentrate in their core. This results in the appearance of strong inhomogeneities in the particle spatial distribution, often dubbed *preferential concentration* [3, 4]. This phenomenon has recently gathered much attention both from a theoretical [5, 6] and a numerical [4, 7] point of view. Progresses in the statistical characterization of particle aggregates have been achieved by studying particles evolving in stochastic flows [5, 8, 9] and in two dimensional turbulent flows [10]. Concerning single particle statistics, there has been considerable less attention for heavy particles [11, 12], in contrast with the numerous studies devoted to tracers [13, 14, 15, 16, 17, 18, 19, 20, 21]. For example, it is known that Lagrangian velocity structure functions of tracers display high intermittent statistics [16, 17, 18], while there are no results concerning heavy particles. The aim of this Letter is to compare the Lagrangian statistics of heavy particles with that of fluid tracers

evolving in the same turbulent flow, by means of Direct Numerical Simulations (DNS). In particular, we shall focus on the Lagrangian structure function of second order, important for stochastic modelling of particle trajectories [22, 23], and of higher orders, to account for intermittency effects. For Lagrangian studies, DNS offer the possibility to reach Reynolds numbers comparable to experiments, with a full control of both temporal and spatial properties of a very large number of tracers and heavy particles.

We consider particles with a mass density ρ_p much larger than the density ρ_f of the carrier fluid. In this limit, particles evolve according to [24]

$$\frac{d\mathbf{X}}{dt} = \mathbf{V}, \quad \frac{d\mathbf{V}}{dt} = -\frac{1}{\tau_s} [\mathbf{V} - \mathbf{u}(\mathbf{X}(t), t)]. \quad (1)$$

$\mathbf{X}(t)$ denotes the particle position, $\mathbf{V}(t)$ its velocity, $\mathbf{u}(\mathbf{x}, t)$ is the fluid velocity and $\tau_s = 2\rho_p a^2 / (9\rho_f \nu)$ is the particle response time, where a is the radius and ν the fluid viscosity. The Stokes number, which quantifies the degree of inertia, is defined as $St = \tau_s / \tau_\eta$, where $\tau_\eta = (\nu / \epsilon)^{1/2}$ is the eddy turnover time associated to the Kolmogorov scale and ϵ the average rate of energy injection. Equation (1) is derived in [24] under the assumption of very dilute suspensions, where particle-particle interactions (e.g. collisions) and hydrodynamic coupling can be neglected. Tracers correspond to the limit $\tau_s \rightarrow 0$, i.e. to the evolution $d\mathbf{X}/dt = \mathbf{u}(\mathbf{X}(t), t)$.

We performed a series of DNS of homogeneous and isotropic turbulence on a cubic grid with resolutions up to 512^3 (reaching a Taylor micro-scale Reynolds number

$R_\lambda \approx 180$) and transporting millions of tracers and particles with $St \in [0.16:1]$. Particles are initially seeded homogeneously in space with velocities equal to the local fluid velocity, already in a stationary configuration. Then they evolve according to equation (1) for about 2 to 3 large-scale eddy turnover times before reaching a Lagrangian statistical steady state. It is only once the particle dynamics has completely relaxed that measurements are started. Details on the simulation parameters can be found in previous reports [11, 25]. For fluid tracers we also present results obtained with resolution 1024^3 and $R_\lambda \approx 300$; these data are described in [17, 21]. The Lagrangian Structure Functions (LSF)

$$S^{(p)}(\tau) = \langle [V(t + \tau) - V(t)]^p \rangle, \quad (2)$$

measure the time variations of any component V of the tracer or particle velocity. For time lags τ in the inertial range (i.e. when $\tau_\eta \ll \tau \ll T_L$, where T_L is the integral Lagrangian time scale), the LSF of tracers are expected to display power law behaviors $S^{(p)}(\tau) \propto \tau^{\zeta(p)}$ (see [16] and references therein). A dimensional estimate derived from the Kolmogorov 1941 theory for Eulerian turbulence predicts $S^{(p)}(\tau) \propto (\epsilon\tau)^{p/2}$. Considerable deviations from the non-intermittent scaling $\zeta(p) = p/2$ were observed in both experiments [16, 18] and DNS [17, 21], with however some disagreement on the actual values of the exponents. For heavy particles there is not any reference theory. As the effect of inertia on the velocity statistics becomes less and less important when increasing τ/τ_s [5], we can guess that particles recover the statistical properties of tracers when $\tau_\eta \sim \tau_s \ll \tau \ll T_L$.

We report in the sequel numerical measurements of the LSF of both particles and tracers for time lags ranging from $\tau_\eta/10$ up to $100\tau_\eta$ of the order of the integral time scale T_L . Here we anticipate the main findings. Comparing the statistical properties of tracers and particles, we find further evidence that the formers are strongly affected by vortex trapping well above the Kolmogorov time scale, i.e. in the range $\tau \in [1:10]\tau_\eta$, while trapping becomes less and less important at increasing St . Trapping events spoil the scaling properties for time lags expected to be inside the inertial range of turbulence. This observation explains the disagreement, pointed out in [18], existing between the scaling behavior of fluid tracers LSF measured in experiments [14, 16] and DNS [17, 21]. We argue that this discrepancy stems from the fact that the experimental LSF scaling exponents are measured in the time range $[3 : 6]\tau_\eta$, exactly where trapping is effective. This leads to a more intermittent statistics than that measured in the inertial range [17, 21].

Figure 1 summarizes the results for the second order LSF at varying Stokes. It should be noted that it is very difficult to identify a power-law scaling range for any Stokes and any time lags. This is evident from the absence of plateaux in the logarithmic derivatives of the

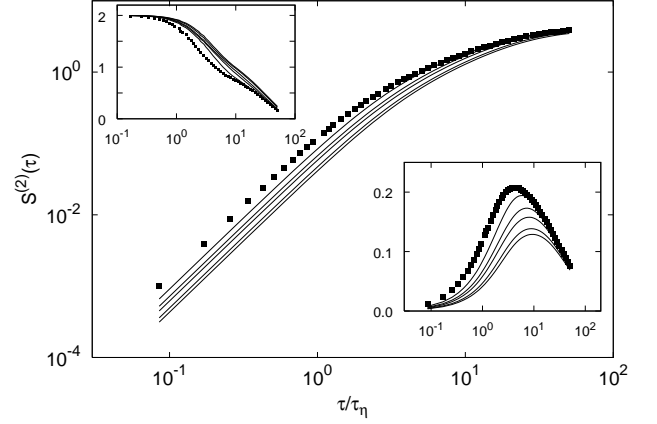


FIG. 1: $S^{(2)}(\tau)$ vs τ/τ_η in log-log scale for tracers (■) and heavy particles with $St = 0.16, 0.37, 0.59, 1.01$, and 1.34 (solid lines from top to bottom). Bottom inset: compensated plot $S^{(2)}(\tau)/\tau$ vs τ/τ_η , same symbols as in the body. Top inset: logarithmic derivative $d \log(S^{(2)}(\tau))/d \log(\tau)$ vs τ/τ_η , same symbols as in the body. For each Stokes number, averages in (2) are performed over $N = 5 \times 10^5$ trajectories that last for about $200\tau_\eta$. All curves are obtained by averaging over the three components of the velocity vector to increase the statistics.

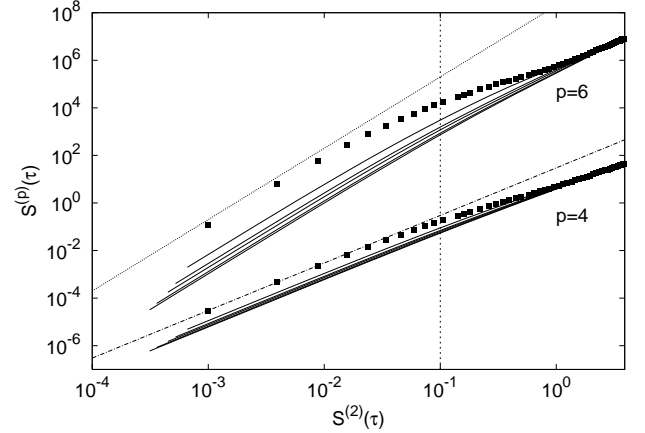


FIG. 2: Log-log plot of $S^{(p)}(\tau)$ vs $S^{(2)}(\tau)$ of order $p = 4$ and $p = 6$ for fluid tracers (■) and heavy particles with $St = 0.16, 0.37, 0.59, 1.01$, and 1.34 (solid lines from top to bottom). The two straight lines represent the dimensional non-intermittent scaling $\zeta(p)/\zeta(2) = p/2$. Notice the similarity of heavy particles for different Stokes and the marked difference of the tracers with respect to particles for values close to the viscous scale, $S^{(2)}(\tau_\eta) \approx 0.1$ (vertical dotted line).

LSF plotted in the top inset. The apparent plateaux in the bottom inset, where we show the compensated LSF normalized to the $S^{(2)}(\tau)/\tau$ at changing Stokes, should thus not be considered as an evidence of a linear power law. Still in the bottom inset, it is worth noticing the strong effects induced by inertia: already for the smallest Stokes number we considered ($St = 0.16$), we

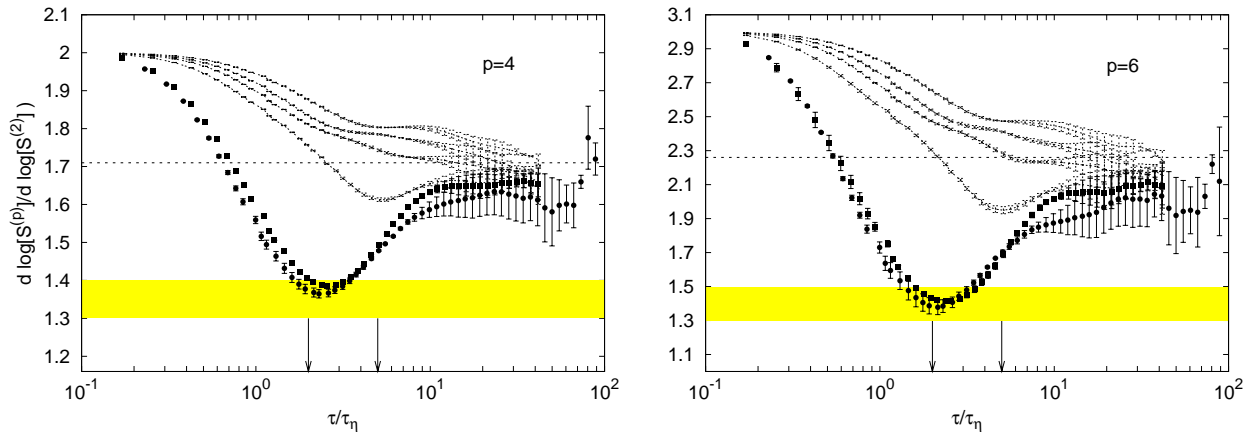


FIG. 3: (Color online) Logarithmic derivative of the ESS plots of Fig. 2 vs τ/τ_η for $p = 4$ (left) and $p = 6$ (right). Data for fluid tracers at $Re_\lambda = 180$ and $Re_\lambda = 300$ are plotted as \blacksquare and \bullet , respectively. The solid lines refer to heavy particles for $St = 0.16, 0.37, 0.59, 1.01$ (from bottom to top). The dotted straight line marks the Lagrangian Multifractal prediction obtained for tracers in [17] ($\zeta(4)/\zeta(2) = 1.71, \zeta(6)/\zeta(2) = 2.26$). The grey (yellow online) band corresponds to the experimental values measured for tracers in [18] at comparable Reynolds numbers, and fitted in the range $\tau \in [2 : 5]\tau_\eta$ (shown with the arrows). Errors have been estimated from the variations among the three velocity components.

observe a clear departure from tracer behavior.

It is now interesting to look for non-trivial effects at higher-order moments. As customary in turbulence studies, assessing deviations from a simple dimensional scaling can be done by comparing all moments against a reference one, a procedure originally proposed for Eulerian structure functions and dubbed Extended Self Similarity (ESS) [26]. This amounts to study the scaling behavior of the order p LSF as a function of that of order 2, used as a reference for Lagrangian statistics. This procedure is known to decrease finite-Reynolds effects and is frequently used for measuring scaling exponents in experiments and simulations. The price to pay is that only relative scaling exponents $\zeta(p)/\zeta(2)$ can be measured. Figure 2 shows $S^{(p)}(\tau)$ as a function of $S^{(2)}(\tau)$ for $p = 4$ and $p = 6$ and various values of St . Two observations can be done. First, the LSF of both inertial particles and tracers have an *inertial-range* scaling behavior that deviates significantly from the dimensional one $\zeta(p)/\zeta(2) = p/2$. Second, the differences between tracers and particles around $\tau \sim \tau_\eta$, i.e. for $S^{(2)}(\tau_\eta) \sim 0.1$, are now even more pronounced. To be more quantitative in Fig. 3 the local slopes $d \log(S^{(p)}(\tau))/d \log(S^{(2)}(\tau))$ of all curves are shown. Let us first focus on the behavior well inside the *inertial range*. For $10\tau_\eta < \tau < T_L$, all the data sets display a tendency to converge, within error bars, to the same scaling behavior as that of fluid tracers, irrespectively of the value of the Stokes number. The large error bars are due to unavoidable large-scale anisotropic fluctuations in the statistics. In the same figure we also report the multifractal prediction for fluid tracers obtained in [17] (see also [13, 27]), by translating the well-known Eulerian multifractal model to the Lagrangian domain.

Notice that the inertial-range behavior is compatible with the multifractal prediction also for heavy particles. For time lags in the range $\tau_\eta \leq \tau \leq 10\tau_\eta$, the local scaling exponents reveal the presence a strong bottleneck effect, much more pronounced for tracers than particles. At these time scales, both for $p = 4$ and $p = 6$, fluid tracers have very intermittent scaling properties that are however smoothed out as soon as inertia is switched on. This is due to the fact that tracers ($St = 0$) may experience vortex trapping lasting for rather long times, while inertial particles (even for $St \ll 1$) are expelled from vortex filaments. The values for the scaling exponents of the LSF given in the experimental study of fluid tracers in [18] at comparable Reynolds numbers are also represented in Fig. 3 in the shaded band. In Ref. [18], the scaling exponents are measured in the range $\tau \in [2 : 5]\tau_\eta$ (see arrows in the figure), where trapping into vortex filaments gives the dominant contribution leading to a more intermittent statistics. For these time lags, the values of the scaling exponents reported in [18] are in good agreement with DNS results. However, these exponents substantially differ from the inertial-range values of the logarithmic derivative that we observe at $\tau > 10\tau_\eta$.

It is worth stressing that the statistical signature of particle trapping into vortex filaments has already been the subject of experimental and numerical studies of fully developed turbulent flows. In particular, it was shown that such events play a crucial role in determining the intense fluctuations of tracer acceleration [14, 16, 17, 28]. This effect has previously been highlighted by filtering out the intense vortical events in the LSF (see Fig. 3 of [21] and compare it with Figs. 1 and 2). Such a filtering is here obtained dynamically by switching on inertia,

whose major landmark at small values of the Stokes number is to expel heavy particles from vortex filaments. In conclusion, we have shown that for large-enough time lags ($\tau > 10\tau_\eta$) the scaling properties of heavy particles with $St \lesssim 1$ tend to be almost independent of St , within error bars. The relative scaling exponents of LSF for the investigated Stokes numbers are found in the range $\zeta(4)/\zeta(2) \in [1.55 : 1.75]$ for the fourth order and $\zeta(6)/\zeta(2) \in [1.8 : 2.4]$ for the sixth one. Investigations at larger Re_λ are needed to confirm the robustness of the universality enjoyed by heavy particles for large time lags. This observation can be explained by the fact that the effect of inertia on the particle velocity should disappear progressively when increasing the scale [5]. For time lags well inside the inertial range and much larger than the response time τ_s , the delay of the particles on the fluid motion becomes negligible and a “tracer-like” physics is recovered [29]. For the case of fluid tracers, it is important to be very careful in assessing scaling properties because trapping into vortex filaments may spoil the *inertial range* scaling behavior for time lags up to $\tau \sim 10\tau_\eta$ and even more. Such trapping effects are less effective for heavy particles due to their dynamical properties that bring them outside of strong vortex filaments. These results are important for stochastic modelling of both tracers and heavy particles, where the statistical properties of velocity and acceleration along the trajectories are the main ingredients for developing appropriate models.

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